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A Geometric Framework for Macroeconomic Analysis

Foundations, Testable Hypotheses, and Implementation

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ABSTRACT
This paper proposes discrete Ricci curvature on financial correlation networks as a geometric indicator of systemic fragility. The framework conceptualizes the economy as a differentiable manifold equipped with a Riemannian metric, where curvature encodes whether small perturbations self-correct (positive curvature) or self-amplify (negative curvature). Building on Mach's relational ontology and Einstein's geometrization program, a testable hypothesis is derived: aggregate network curvature is associated with financial stress and serves as an indicator of systemic fragility. Dimensional homogeneity is addressed through non-dimensionalization, candidate conservation principles are proposed, and a complete worked example is provided. The relationship between continuum curvature of the state manifold and discrete curvature of correlation networks is clarified against prior empirical findings. An implementation architecture enables empirical validation. The framework is offered as a research program rather than a finished theory.

KEYWORDS geometric economics; Riemannian manifolds; Ricci curvature; financial crises; economic networks; information geometry

Contents

1 Introduction

The application of differential geometry to economic systems has a long but discontinuous history. From Irving Fisher's explicit adoption of thermodynamic concepts (Mirowski, 1989) through Samuelson's exploration of revealed preference geometry (Samuelson, 1950) to recent work on gauge-theoretic economics (Malaney, 1996; Smolin, 2009; Vázquez and Farinelli, 2009), the geometric approach has produced isolated insights without coalescing into a unified framework. More recently, discrete Ricci curvature has been applied directly to financial correlation networks as an indicator of systemic risk (Sandhu et al., 2016; Samal et al., 2021). These network diagnostics have proven empirically fruitful but remain disconnected from any underlying geometric theory of the economic state space; supplying that missing foundational layer is the aim of the present paper.

This paper proposes such a unifying framework, not as a finished theory, but as a *minimal foundational structure* upon which specific models can be built, tested, and refined. The approach presented here synthesizes two intellectual traditions: Ernst Mach's relational ontology and Albert Einstein's geometric theory of gravitation. This synthesis addresses specific limitations of conventional economic modeling while remaining empirically grounded.

1.1 Scope and Claims

Before proceeding, it is important to clarify what this paper does and does not claim:

What is claimed:

- Discrete Ricci curvature on financial networks captures structural fragility not visible in simple correlation statistics
- Curvature provides a geometric interpretation of systemic risk that complements existing measures
- The framework generates testable hypotheses about the relationship between network geometry and financial stress

What is not claimed:

- That curvature definitively *predicts* crises with positive lead time (this is an empirical question)
- That the economy literally *is* a curved spacetime
- That this framework supersedes existing approaches to systemic risk

The paper makes three contributions. First, a minimal axiomatic framework is articulated (Section 2) providing a common language for geometric economics. Second, testable hypotheses are derived (Section 6) connecting manifold curvature to financial crisis dynamics, including an explicit reconciliation of continuum and discrete curvature sign conventions. Third, an implementation architecture is specified (Section 8) enabling empirical validation.

1.2 The Problem with Neoclassical Foundations

Modern macroeconomics rests on foundations that are increasingly recognized as problematic (Lawson, 2003). Two issues are particularly relevant for present purposes.

The Absolutism of Value. Neoclassical economics treats prices and values as properties of individual goods, modified by supply and demand but ultimately grounded in intrinsic utility or production costs. This absolutism, the idea that value exists independently of the entire system of economic relations, mirrors the Newtonian conception of absolute space and time.

The Flatness Assumption. Standard models implicitly assume that the "space" of economic possibilities is flat, that is, Euclidean. Optimization problems are solved as if agents navigate a homogeneous landscape where the distance between any two states is simply the sum of component differences. This ignores the possibility that economic space is curved: that some transitions are systematically harder than others, and that the structure of economic space itself varies across regions and time.

These two problems, absolutism and flatness, are precisely what the Mach-Einstein synthesis addresses. Mach dissolves absolutism; Einstein introduces curvature.

1.3 Ernst Mach: The Relational Turn

Ernst Mach (1838–1916) was an Austrian physicist and philosopher whose critique of Newtonian mechanics profoundly influenced Einstein's development of relativity. Mach's central insight was that the concept of "absolute space", a fixed, immovable background against which all motion is measured, was metaphysically unjustified and empirically empty.

> "No one is competent to predicate things about absolute space and absolute motion; they are pure things of thought, pure mental constructs, that cannot be produced in experience." (Mach, 1883); English translation from Mach (1919, p. *(to be supplied)*)

For Mach, all meaningful physical statements must be relational: the motion of a body can only be defined relative to other bodies, not relative to an invisible, undetectable absolute frame.

The economic translation is direct: *no economic magnitude has intrinsic meaning independent of its relations to all other magnitudes in the system*. A price is not a property of a good but a relation between goods, expectations, institutions, and the entire configuration of the economy.

This Machian perspective implies:

1. **No Absolute Numéraire**: The choice of numéraire (dollar, gold, labor-hour) is purely conventional; no unit of account has privileged status.

2. **Holistic Determination**: Each economic variable is determined by all others simultaneously; partial equilibrium analysis is at best an approximation.

3. **Path Dependence**: Since relations constitute values, changing the path through economic space can change the values themselves.

The Malaney-Weinstein work on gauge theory and the index number problem (Malaney, 1996) can be understood as a rigorous implementation of Machian economics: price indices transform under numéraire change exactly like gauge potentials transform under gauge transformation.

1.4 Albert Einstein: Geometry as Physics

Albert Einstein (1879–1955) transformed Mach's philosophical critique into a revolutionary physical theory. In General Relativity (1915), Einstein showed that gravitation is not a force acting across absolute space but a manifestation of spacetime curvature. Mass-energy determines the geometry of spacetime; geometry determines how objects move.

Several features of Einstein's approach are relevant for economics:

Intrinsic Geometry. Riemannian geometry describes curved spaces "from the inside," without reference to an embedding in higher-dimensional flat space. This is crucial for economics: one cannot step outside the economy to view it from an absolute vantage point.

Curvature as Information. The curvature tensor encodes how parallel transport around a closed loop fails to return a vector to its original orientation. Positive curvature makes nearby paths converge; negative curvature makes them diverge. In economic terms, curvature could encode whether small perturbations are self-correcting (positive) or self-amplifying (negative).

Local vs. Global. General Relativity is a local theory, yet local curvature has global consequences. Similarly, economic disturbances may be local in origin but global in effect.

The transfer from physics to economics is not merely metaphorical. The mathematical apparatus of Riemannian geometry (metrics, connections, curvature tensors) is a general framework for describing any space with a notion of distance. That economic systems possess such structure is a hypothesis about appropriate mathematical language, not a claim about deep ontological similarity to physical spacetime.

1.5 Epistemological Cautions

The transfer of physical concepts to economics carries well-documented risks. Philip Mirowski's *More Heat than Light* (Mirowski, 1989) demonstrated how neoclassical economics borrowed mathematical formalism from 19th-century energetics without the underlying conservation laws that justified it.

I am acutely aware of this danger. Several points of caution are essential:

Mathematical Isomorphism $\neq$ Ontological Identity. That economic systems can be described using Riemannian geometry does not mean economies *are* curved spacetimes. The same mathematical structure can describe radically different phenomena.

Conservation Laws. In General Relativity, the covariant divergence of the stress-energy tensor vanishes: $\nabla_\mu T^{\mu\nu} = 0$. This encodes energy-momentum conservation. What is the economic analogue? This question is addressed directly in Section 4, where candidate conservation principles are proposed.

Falsifiability. A theory that can accommodate any observation explains nothing. The geometric framework must generate testable predictions that could, in principle, be refuted. This is attempted in Section 6.

Dimensional Consistency. Physical equations must be dimensionally consistent. Economic variables have diverse units. This is addressed in Section 3, showing how to construct a well-defined metric from dimensionless quantities.

1.6 Structure of the Paper

The remainder of the paper proceeds as follows:

Section 2 presents the formal framework: two axioms, the definition of the economic manifold, and the core geometric objects.

Section 3 addresses dimensional homogeneity, showing how to construct a mathematically well-defined metric.

Section 4 proposes candidate conservation principles for geometric economics.

Section 5 provides a complete worked example: a two-sector economy with explicit metric, curvature calculation, and geodesics.

Section 6 derives testable hypotheses connecting manifold curvature to financial dynamics and reconciles continuum and discrete sign conventions.

Section 7 develops the Fisher information metric as a principled specification.

Section 8 specifies a software architecture for empirical validation.

Section 9 compares geometric economics to existing approaches.

Section 10 discusses limitations and concludes the paper.

2 Theoretical Framework

2.1 Foundational Axioms

The framework rests on two axioms that constrain but do not fully determine the resulting theory.

Axiom 1 (Relational Ontology)**.** No economic state possesses intrinsic properties independent of its relations to all other states in the system.

This axiom, inspired by Mach's critique of Newtonian absolute space, implies that prices, values, and productivities are fundamentally relational.

Axiom 2 (Geometric Structure)**.** The economy constitutes a differentiable manifold $\mathscr{M}$ equipped with a Riemannian metric $g_{\mu\nu}$.

This axiom asserts that the space of economic states has sufficient structure for differential calculus—continuity, differentiability, and a notion of distance.

Note on Extensibility: The manifold $\mathscr{M}$ may include coordinates beyond directly observable market variables, representing institutional quality, informational state, or expectational coordination. This is treated as a methodological option rather than a foundational axiom; the core framework operates with observable coordinates, with extensions available as the research program develops.[1]

2.2 The Economic Manifold

Definition 1 (Economic Manifold)**.** The economic manifold is a tuple $(\mathscr{M}, g)$ where:

- $\mathscr{M}$ is an n-dimensional smooth manifold
- $g_{\mu\nu} : T_p\mathscr{M} \times T_p\mathscr{M} \to \mathbb{R}$ is a Riemannian metric tensor
- Each point $p \in \mathscr{M}$ represents a complete macroeconomic state

In a minimal specification with n assets:

$$p = (r^1, r^2, \ldots, r^n) \tag{1}$$

where r^i represents the log-return of asset i (dimensionless).

2.3 The Metric: Economic Distance

The metric tensor $g_{\mu\nu}(p)$ defines the infinitesimal "economic distance" between neighboring states:

$$ds^2 = g_{\mu\nu}(p)\, dx^\mu dx^\nu \tag{2}$$

Definition 2 (Economic Distance)**.** The quantity ds represents the total "cost" or "resistance" of transitioning from state p to state $p + dp$. This cost aggregates multiple friction sources into a single geometric measure.

[1] The idea that complete system description may require "hidden" coordinates encoding organizational structure has precedents in physics, though no commitment is made to any specific dimensional scheme.

The framework is agnostic about the specific form of $g_{\mu\nu}$. Possible specifications include:

1. **Fisher Information Metric**: Derived from statistical distinguishability of economic states (see Section 7); the preferred specification.
2. **Correlation-Based Metric**: Derived from asset return correlations, where distance inversely relates to correlation.
3. **Phenomenological Metric**: Calibrated directly from empirical data on transaction costs and market frictions.

2.4 Curvature: Systemic Interdependence

From the metric, the Christoffel symbols are derived:

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right) \tag{3}$$

The Riemann curvature tensor:

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda} \Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda} \Gamma^{\lambda}_{\mu\sigma} \tag{4}$$

The Ricci tensor and scalar curvature:

$$R_{\mu\nu} = R^{\rho}_{\mu\rho\nu}, \qquad R = g^{\mu\nu} R_{\mu\nu} \tag{5}$$

Remark 1 (Curvature Interpretation)**.** The scalar curvature R summarizes the local geometry of economic space:

- $R > 0$: Nearby trajectories tend to converge; perturbations tend to self-correct
- $R < 0$: Nearby trajectories tend to diverge; perturbations tend to amplify
- $R = 0$: Holds in flat space, the idealized limit of independent, non-interacting agents

This dictionary is exact in two dimensions, where the scalar curvature determines the geometry completely. In higher dimensions geodesic deviation is governed by sectional curvature, that is, by directional contractions of the Riemann tensor, and the scalar curvature is only an aggregate contraction: a manifold may exhibit converging geodesics in some directions and diverging geodesics in others at the same point. Zero scalar curvature likewise does not imply flatness in general, since a manifold can be scalar-flat while retaining non-zero Ricci or Riemann curvature. The interpretation above is therefore a heuristic summary statistic in dimensions greater than two, not a complete characterization of stability.

2.5 Geodesics: Paths of Least Resistance

Definition 3 (Economic Geodesic)**.** A geodesic on $(\mathscr{M},g)$ is a curve $x^\mu(s)$ satisfying:

$$\frac{d^2x^\mu}{ds^2}+\Gamma^\mu_{\nu\rho}\frac{dx^\nu}{ds}\frac{dx^\rho}{ds}=0 \tag{6}$$

Geodesics are locally length-extremizing curves; under suitable conditions they are locally friction-minimizing, though a geodesic need not be the globally cheapest path between two states. They are interpreted as the trajectories an economic system would follow if agents collectively minimized transition costs, given current constraints encoded in the metric. Deviations from geodesics require additional "force": policy intervention, technological shocks, or coordination failures.

Important clarification: In an economy, all actions involve agents making decisions; there is no "external" intervention in the strict sense. "Deviation from geodesic" is interpreted as deviation from the path that would minimize aggregate friction given the *current* institutional structure. Policy changes, for example, alter the metric itself rather than simply pushing the economy off a geodesic.

2.6 The Minimal Framework: Summary

Table 1: Core Components of the Geometric Framework

Component	Mathematical Object	Economic Interpretation
State space	Manifold $\mathscr{M}$	Space of macroeconomic configurations
Distance	Metric $g_{\mu\nu}$	Aggregate transition friction
Interdependence	Curvature $R_{\mu\nu\rho\sigma}$	Self-correcting vs. self-amplifying dynamics
Efficient paths	Geodesics	Friction-minimizing trajectories

3 Dimensional Homogeneity and Non-Dimensionalization

A rigorous geometric treatment requires that the metric $ds^2=g_{\mu\nu}dx^\mu dx^\nu$ be dimensionally consistent. Economic variables have heterogeneous units (dollars, hours, quantities), so naïvely constructing a metric from raw variables is mathematically ill-defined.

3.1 The Problem

Consider two coordinates: price P (in dollars) and quantity Q (in units). The expression $ds^2=dP^2+dQ^2$ mixes incompatible dimensions. What does it mean to add dollars-squared to units-squared?

3.2 The Solution: Dimensionless Coordinates

The approach taken here works exclusively with **dimensionless** quantities:

1. **Log-returns**: For prices $P_i(t)$, define

$$r_i(t) = \ln\left(\frac{P_i(t)}{P_i(t-1)}\right) \tag{7}$$

   Log-returns are dimensionless (pure numbers) and symmetric for gains/losses.

2. **Ratios**: For any extensive quantity X, work with X/X_0 where X_0 is a reference value (e.g., mean, initial value, or market total).

3. **Z-scores**: Standardize variables to zero mean and unit variance:

$$z_i = \frac{X_i - \bar{X}}{\sigma_X} \tag{8}$$

3.3 The Correlation-Based Metric

For financial applications, distances are constructed from return correlations. Let ρ_{ij} be the correlation between log-returns of assets i and j. Define the distance introduced by Mantegna (1999):

$$d_{ij} = \sqrt{2(1-\rho_{ij})} \tag{9}$$

This satisfies:

- $d_{ij} = 0$ when $\rho_{ij} = 1$ (perfectly correlated assets are "at the same point")
- $d_{ij} = 2$ when $\rho_{ij} = -1$ (perfectly anti-correlated assets are maximally distant)
- $d_{ij} = \sqrt{2}$ when $\rho_{ij} = 0$ (uncorrelated assets are at intermediate distance)
- Triangle inequality is satisfied

This construction yields a finite metric space on the set of assets rather than a metric tensor on a continuum. It is therefore used directly at the discrete level, as edge weights of the financial network on which discrete curvature is computed (Section 6). A continuum metric tensor consistent with the same data can be obtained, when needed, from the Fisher information construction of Section 7, which is the principled route to $g_{\mu\nu}$.

3.4 Dimensional Consistency Check

With dimensionless coordinates r^i:

- dr^i is dimensionless

- g_{ij} is dimensionless
- $ds^2 = g_{ij} dr^i dr^j$ is dimensionless
- Curvature R has dimensions of (length)$^{-2}$, but since the "length" is dimensionless, R is also dimensionless

The framework is now mathematically well-defined.

4 Candidate Conservation Principles

Einstein's field equations derive their power from energy-momentum conservation: $\nabla_\mu T^{\mu\nu} = 0$. Without an analogous conservation law, geometric economics risks being empty formalism. Three candidates are proposed here, with the acknowledgment that this remains an open problem.

4.1 Candidate 1: Accounting Identities

National income accounting provides exact conservation laws:

$$Y \equiv C + I + G + (X - M) \tag{10}$$

Income equals expenditure by definition. At the sectoral level:

$$(S - I) + (T - G) + (M - X) \equiv 0 \tag{11}$$

Private, government, and foreign balances sum to zero.

Geometric interpretation: These identities constrain the manifold's structure. Not all points in "coordinate space" correspond to valid economic states; the economy must lie on a submanifold satisfying accounting constraints. This is analogous to how gauge constraints in physics restrict dynamics to a subspace.

4.2 Candidate 2: No-Arbitrage Conditions

In financial markets, absence of arbitrage provides differential constraints. By the fundamental theorem of asset pricing, a market is arbitrage-free precisely when there exists an equivalent martingale measure $\mathbb{Q}$ under which discounted asset prices are martingales; for an asset S_t this takes the familiar form

$$dS_t = rS_t\,dt + \sigma S_t\,dW_t^{\mathbb{Q}} \tag{12}$$

where r is the risk-free rate and $W_t^{\mathbb{Q}}$ is a Brownian motion under $\mathbb{Q}$; with a continuous dividend yield q the drift becomes $r - q$.

Vázquez and Farinelli (2009) showed that arbitrage corresponds to *curvature* of a gauge connection. In a no-arbitrage market, certain curvature components vanish. This directly connects financial constraints to geometry.

Geometric interpretation: No-arbitrage is a “flatness” condition on part of the economic manifold. Violations of no-arbitrage correspond to non-zero curvature in specific directions.

4.3 Candidate 3: Budget Constraints and Walras’ Law

Individual budget constraints aggregate to Walras’ Law: the sum of excess demands across all markets is identically zero:

$$\sum_i p_i z_i(p) \equiv 0 \tag{13}$$

Geometric interpretation: This is a constraint on the “flow” of economic activity, analogous to conservation of current in electromagnetism. It does not constrain all dynamics but ensures consistency of the price system.

4.4 The Status of Conservation in This Framework

It is not claimed here that the conservation problem has been solved. Rather, the proposal is:

1. Accounting identities provide **exact** constraints that define the economic submanifold
2. No-arbitrage conditions provide **approximate** constraints in financial markets
3. Budget constraints provide **aggregate** flow conservation

A complete geometric economics would derive field equations from these conservation principles. This remains an open problem for future theoretical development.

5 Worked Example: A Two-Sector Economy

To demonstrate that the framework produces concrete, calculable results, a complete example is presented: a two-dimensional economic manifold with explicit metric, curvature, and geodesics.

5.1 Setup

Consider an economy with two sectors, represented by log-returns (r^1, r^2). A metric is posited where sector 1’s state affects the cost of adjusting sector 2:

$$ds^2 = (dr^1)^2 + e^{\alpha r^1}(dr^2)^2 \tag{14}$$

where $\alpha > 0$ is a parameter controlling curvature intensity. This metric captures the asymmetric interdependence: when sector 1 has positive returns ($r^1 > 0$), adjustments in sector 2 become more "costly" (larger metric coefficient); when sector 1 has negative returns, sector 2 adjustments become "cheaper."

In matrix form:

$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & e^{\alpha r^1} \end{pmatrix} \tag{15}$$

Note on metric choice: A naïve product metric $ds^2 = f_1(r^1)(dr^1)^2 + f_2(r^2)(dr^2)^2$ would be flat (zero curvature) in 2D. Genuine curvature requires coupling between coordinates, as in the metric above where g_{22} depends on r^1.

5.2 Christoffel Symbols

For this metric with $g_{11} = 1$ and $g_{22} = e^{\alpha r^1}$, the non-zero Christoffel symbols are:

$$\Gamma^1_{22} = -\frac{1}{2g_{11}} \frac{\partial g_{22}}{\partial r^1} = -\frac{\alpha}{2} e^{\alpha r^1} \tag{16}$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{2g_{22}} \frac{\partial g_{22}}{\partial r^1} = \frac{\alpha}{2} \tag{17}$$

5.3 Gaussian Curvature

For a 2D metric of the form $ds^2 = (dr^1)^2 + h(r^1)(dr^2)^2$ with $h = e^{\alpha r^1}$, the Gaussian curvature is:

$$K = -\frac{1}{\sqrt{h}} \frac{d^2\sqrt{h}}{d(r^1)^2} \tag{18}$$

Computing:

$$\sqrt{h} = e^{\alpha r^1/2} \tag{19}$$

$$\frac{d\sqrt{h}}{dr^1} = \frac{\alpha}{2} e^{\alpha r^1/2} \tag{20}$$

$$\frac{d^2\sqrt{h}}{d(r^1)^2} = \frac{\alpha^2}{4} e^{\alpha r^1/2} \tag{21}$$

Therefore:

$$K = -\frac{1}{e^{\alpha r^1/2}} \cdot \frac{\alpha^2}{4} e^{\alpha r^1/2} = -\frac{\alpha^2}{4} \tag{22}$$

The scalar curvature in 2D is $R = 2K = -\frac{\alpha^2}{2}$.

5.4 Interpretation

The curvature is **constant and negative** throughout the manifold:

- $K = -\alpha^2/4 < 0$ for any $\alpha \neq 0$
- Larger α (stronger coupling) produces more negative curvature
- Negative curvature means nearby geodesics diverge; small perturbations amplify

Economic interpretation: The asymmetric coupling between sectors creates inherent instability. When sector 1 moves, it changes the "landscape" for sector 2, and this interdependence produces divergent dynamics characteristic of negative curvature. This is a minimal model of how sectoral coupling can generate systemic fragility.

5.5 Geodesic Equations

The geodesic equations are:

$$\frac{d^2r^1}{ds^2} - \frac{\alpha}{2}e^{\alpha r^1}\left(\frac{dr^2}{ds}\right)^2 = 0 \tag{23}$$

$$\frac{d^2r^2}{ds^2} + \alpha\frac{dr^1}{ds}\frac{dr^2}{ds} = 0 \tag{24}$$

The second equation can be integrated: if $u = dr^2/ds$, then $du/ds = -\alpha u(dr^1/ds)$, giving $u = Ce^{-\alpha r^1}$ for some constant C. This shows that motion in the r^2 direction is suppressed as r^1 increases—geodesics "straighten out" in the direction of increasing sector 1 returns.

5.6 Summary

This toy model demonstrates:

1. The framework produces explicit, calculable geometric quantities
2. Cross-coordinate coupling is necessary for non-trivial curvature in 2D
3. Constant negative curvature emerges from asymmetric sectoral interdependence
4. Geodesics can be computed and show economically interpretable behavior

Real applications will use empirically calibrated metrics, but the mathematical machinery is identical.

6 Testable Hypotheses: Curvature and Financial Dynamics

6.1 From Framework to Prediction

In the continuum framework, regions of negative curvature represent zones where small perturbations amplify. Operationalizing this intuition on observed data requires care, since the discrete curvature of a correlation network and the continuum curvature of the underlying state manifold are related but distinct objects, and prior empirical work shows that their sign conventions differ (Section 6.3). The framework therefore generates a structural claim, that curvature carries fragility information beyond simple statistics, together with sign expectations that depend on which curvature notion is employed.

6.2 Operationalization via Discrete Ricci Curvature

Direct computation of continuous Riemannian curvature requires a fully specified metric. The curvature concept can nonetheless be operationalized through *discrete Ricci curvature* on financial networks, a technique first developed for biological networks (Sandhu et al., 2015) and subsequently applied to financial markets (Sandhu et al., 2016).

Definition 4 (Financial Correlation Network)**.** Let $G=(V,E,w)$ be a weighted graph where:

- $V=\{1,\dots,N\}$ represents N financial assets
- $E\subseteq V\times V$ represents significant correlations
- $w_{ij}=\sqrt{2(1-\rho_{ij})}$ is the correlation-based distance

Definition 5 (Ollivier-Ricci Curvature (Ollivier, 2009))**.** The Ollivier-Ricci curvature of edge (i,j) is:

$$\kappa(i,j)=1-\frac{W_1(\mu_i,\mu_j)}{d(i,j)} \tag{25}$$

where W_1 is the Wasserstein-1 distance and μ_i, μ_j are probability measures on the neighborhoods of i and j.

Interpretation: Ollivier-Ricci curvature compares the “transport cost” between neighborhoods to the direct edge distance. Positive curvature indicates the neighborhoods are closer than the nodes themselves (clustered structure); negative curvature indicates the edge is a “bottleneck” connecting otherwise distant regions.

Definition 6 (Forman-Ricci Curvature (Forman, 2003; Sreejith et al., 2016))**.** For an unweighted graph, the Forman-Ricci curvature of edge (i,j) is

$$F(i,j)=4-\deg(i)-\deg(j), \tag{26}$$

with the augmented variant adding a contribution of $+3$ for every triangle containing the edge; weighted generalizations incorporate node and edge weights (Samal et al., 2018).

Interpretation: Triangle terms belong to the augmented variant, computed on the 2-dimensional simplicial complex including triangular faces, not to the ordinary 1-dimensional Forman curvature. Under the augmented variant, edges participating in many triangles tend to have higher curvature, indicating local clustering; under either variant, edges serving as bridges between otherwise disconnected regions have strongly negative curvature, indicating structural bottlenecks. All Forman-Ricci computations in this paper use the augmented variant, which is also the default of the reference implementation (Section 8).

Recent work suggests Forman-Ricci may have superior empirical properties for some applications (Samal et al., 2018). It is recommended to compute both measures for robustness.

6.3 Sign Conventions and Prior Evidence

Continuum and discrete curvature point in superficially opposite directions, and this tension must be addressed openly. In the continuum picture, negative curvature of the state manifold means diverging geodesics and self-amplifying perturbations, so fragility corresponds to negative curvature. In the discrete picture, Sandhu et al. (2016) found that the average Ollivier-Ricci curvature of equity correlation networks rises sharply during crisis periods, and Samal et al. (2021) identified extreme curvature values as hallmarks of market crashes. The discrete result is driven by correlation densification: in a crisis, assets move together, the network becomes tightly clustered, and clustering produces positive edge curvature.

There is no contradiction once the two objects are distinguished. Discrete curvature of the realized correlation network measures the clustering and bottleneck structure of the dependence graph, while continuum curvature of the underlying manifold measures the stability of perturbation dynamics. A tightly clustered network (high discrete curvature) reflects a collapse of effective dimensionality and a loss of diversification, which is precisely the condition under which the continuum dynamics become unstable. High discrete curvature of the dependence graph and negative continuum curvature of the state manifold are thus complementary signatures of the same fragile regime. To be explicit: no claim is made that the discrete graph curvature is a numerical discretization of the continuum scalar curvature of Section 2; establishing such a correspondence would require an explicit embedding, reconstruction method, or convergence argument, and this remains an open theoretical problem. The hypothesis below is accordingly stated in terms of association and incremental information, with the sign for each curvature measure treated as a per-measure empirical regularity rather than a universal prediction.

6.4 Formal Hypothesis Statement

Hypothesis 1 (Curvature-Fragility Relationship)**.** Let $\bar{\kappa}(t)$ be the aggregate Ricci curvature of a financial correlation network at time t. Then:

1. **Association with stress**: Deviations of $\bar{\kappa}(t)$ from its regime baseline are associated with contemporaneous measures of financial stress (VIX, credit spreads, drawdowns). Following prior evidence, the expected sign for Ollivier-Ricci curvature on correlation networks is positive: $\bar{\kappa}(t)$ rises with stress.

2. **Structural information**: Curvature captures network fragility not explained by simple correlation statistics (average correlation, network density).

3. **Potential leading indicator**: Curvature *may* shift before stress events, but this is an empirical question requiring careful testing.

Note on predictive claims: Existing evidence positions curvature as a "crash hallmark," a contemporaneous indicator, rather than a leading indicator with positive lead time (Samal et al., 2021). The hypothesis is framed conservatively: curvature captures structural fragility, and whether this fragility precedes or accompanies crises is to be determined empirically. Either result is scientifically valuable.

6.5 Empirical Predictions

1. **Cross-sectional**: At any point in time, sectors/assets whose local curvature deviates most strongly from baseline should have higher subsequent volatility.

2. **Time-series**: Aggregate curvature should correlate with realized financial stress measures.

3. **Incremental value**: Curvature should explain variance in stress not captured by simpler network statistics.

4. **Regime dependence**: The curvature-stress relationship may be stronger during high-volatility regimes.

7 The Fisher Information Metric

The Fisher information metric provides a principled, non-arbitrary specification of $g_{\mu\nu}$, connecting the geometric framework to information geometry.

7.1 Construction

Let economic states be parameterized by $\theta = (\theta^1, \ldots, \theta^n)$, and let $p(x|\theta)$ be the probability distribution of observable outcomes x given state θ. The Fisher information matrix is:

$$g_{ij}(\theta) = \mathbb{E}\left[\frac{\partial \log p(x|\theta)}{\partial \theta^i} \cdot \frac{\partial \log p(x|\theta)}{\partial \theta^j}\right] \tag{27}$$

This defines a Riemannian metric with line element:

$$ds^2 = g_{ij}(\theta) d\theta^i d\theta^j \tag{28}$$

7.2 Uniqueness: Chentsov's Theorem

The Fisher metric is distinguished by **Chentsov's theorem** (Čencov, 1982; Amari and Nagaoka, 2000): under specified invariance conditions, it is the unique Riemannian metric, up to a constant multiple, on the relevant class of statistical models that is invariant under sufficient statistics. Three qualifications matter. Chentsov-type uniqueness results hold for defined classes of statistical models (finite sample spaces in the original theorem, with extensions to parametric families under additional regularity conditions), not for arbitrary statistical manifolds. Invariance under sufficient statistics does not make every modelling choice or finite-sample representation lossless; the theorem fixes the metric given the model, it does not validate the model. Within these limits, the choice of metric is not arbitrary but forced by invariance requirements, which provides a strong theoretical foundation for the specification.

7.3 Example: Multivariate Gaussian

For returns $r \sim \mathcal{N}(\mu, \Sigma)$ with parameters $\theta = (\mu, \Sigma)$, the Fisher metric on the mean parameters (holding covariance fixed) is:

$$g_{ij} = (\Sigma^{-1})_{ij} \tag{29}$$

The inverse covariance matrix, the precision matrix, is the natural metric. This has an intuitive interpretation: directions of high precision (low variance) have large metric coefficients, meaning small changes in those directions are "costly" in information-theoretic terms.

7.4 Curvature of Gaussian Manifolds

For a univariate Gaussian with $\theta = (\mu, \sigma)$:

$$g = \begin{pmatrix} \sigma^{-2} & 0 \\ 0 & 2\sigma^{-2} \end{pmatrix} \tag{30}$$

The Gaussian curvature is constant and negative, $K = -1/2$, with scalar curvature $R = 2K = -1$. The negative curvature indicates that statistical manifolds of Gaussian distributions are intrinsically hyperbolic; small parameter changes lead to diverging probability distributions.

For multivariate Gaussians, curvature depends on the covariance structure and can vary across the manifold.

7.5 Connection to Economic Distance

The Fisher metric interprets economic distance as **statistical distinguishability**:

- Two states are "close" if their observable distributions are hard to distinguish
- Two states are "far" if observations readily discriminate between them

This connects naturally to information economics and rational inattention: agents face costs proportional to the statistical distance between the distributions they must distinguish.

7.6 Implementation Path

A Fisher-metric implementation requires:

1. Specify the parametric family $p(x|\theta)$ (e.g., multivariate Gaussian, mixture models)
2. Estimate parameters $\theta(t)$ at each time point
3. Compute the Fisher information matrix $g_{ij}(\theta(t))$
4. Derive curvature from the metric

This is more demanding than the correlation-based approach but provides stronger theoretical grounding.

8 Implementation Architecture

A modular software architecture is specified here for testing the hypotheses and supporting future extensions.

8.1 System Overview

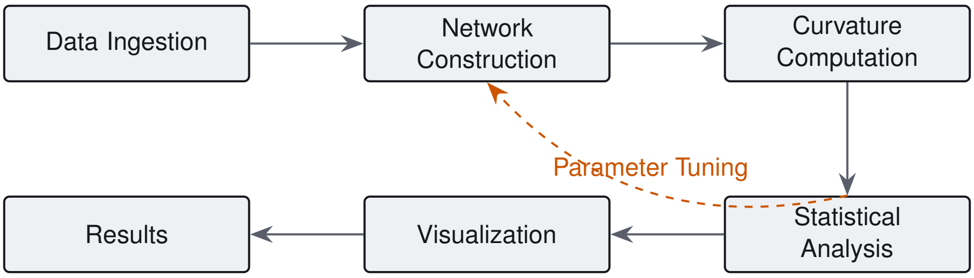

Figure 1: System Architecture

8.2 Existing Tools

The discrete curvature computations can leverage existing implementations:

- **GraphRicciCurvature** (Python): Implements Ollivier-Ricci and Forman-Ricci curvature for NetworkX graphs; for Forman-Ricci it distinguishes the `1d` and `augmented` methods and defaults to `augmented`, the variant adopted throughout this paper

- **POT** (Python Optimal Transport): Efficient Wasserstein distance computation (Flamary et al., 2021)
- **NetworkX / graph-tool**: Network construction and analysis (Hagberg et al., 2008)

8.3 Algorithm: Rolling Window Curvature

Algorithm 1 Rolling Window Network Curvature

Require: Price matrix $P \in \mathbb{R}^{T \times N}$, window size w, threshold θ
Ensure: Time series of aggregate curvatures $\{\bar{\kappa}_t\}$

for $t = w$ to T **do**
  $R_t \leftarrow \text{LogReturns}(P[t-w:t,:])$
  $\rho_t \leftarrow \text{CorrelationMatrix}(R_t)$
  $D_t \leftarrow \sqrt{2(1-\rho_t)}$
  $G_t \leftarrow \text{ThresholdedGraph}(\rho_t, D_t, \theta)$
  $\kappa_t \leftarrow \text{OllivierRicciCurvature}(G_t)$
  $\bar{\kappa}_t \leftarrow \text{Mean}(\kappa_t)$
end for
return $\{\bar{\kappa}_t\}$

8.4 Technology Stack

Table 2: Recommended Technology Stack

Component	Technology	Rationale
Language	Python 3.10+	Ecosystem, scientific computing
Data	pandas, numpy	Standard tools
Networks	NetworkX, graph-tool	Curvature algorithms available
Curvature	GraphRicciCurvature	Specialized library
Transport	POT	Wasserstein distance
Visualization	matplotlib, plotly	Interactive dashboards
Statistics	scipy, statsmodels	Hypothesis testing

9 Comparison to Existing Approaches

9.1 What Does Geometric Economics Add?

Compared to network economics: Standard network measures (degree, centrality, clustering) describe topology but not geometry. Curvature integrates local and global structure, capturing whether perturbations amplify or dissipate, information not contained in topological statistics.

Compared to correlation analysis: Simple correlation measures (average correlation, corre-

Table 3: Geometric Economics vs. Alternative Frameworks

Approach	Ontology	Mathematics	Crisis Mechanism	Prediction
Standard DSGE	Equilibrium	Optimization	Exogenous shocks	None (ex-post)
Network Economics	Relational	Graph theory	Contagion	Centrality
Agent-Based	Emergent	Simulation	Cascades	Distribution tails
Gauge-Theoretic	Relational	Fiber bundles	Arbitrage breakdown	Connection curvature
This paper	Relational	Riemannian geometry	Geometric instability	Ricci curvature

lation dispersion) miss the *structure* of correlations. Curvature detects “bottlenecks” and fragile bridges that correlation statistics cannot see.

Compared to gauge-theoretic economics: The gauge approach focuses on arbitrage and price consistency; the approach presented here focuses on systemic stability. These are complementary: gauge curvature detects arbitrage opportunities; Ricci curvature detects structural fragility.

Compared to existing curvature diagnostics: Sandhu et al. (2016) and Samal et al. (2021) apply discrete Ricci curvature to market data as a standalone empirical indicator. The present framework embeds that indicator in a foundational structure (axioms, metric specification via Fisher information, candidate conservation principles) from which the indicator can be derived rather than merely posited.

The “so what?” answer: Geometric economics provides a *unified mathematical language* linking local network structure to global dynamics via established machinery (Riemannian geometry). It generates predictions (curvature as fragility indicator) that emerge from the formalism rather than being imposed ad hoc.

10 Discussion and Conclusion

The framework presented in this paper offers a geometrically grounded approach to measuring systemic fragility in financial networks. By applying discrete Ricci curvature to correlation-based network structures, the implementation provides a quantitative indicator that captures structural vulnerabilities not visible through conventional correlation statistics or simple network topology measures.

Several aspects of the framework merit careful consideration when deploying this approach in practice. The choice of metric specification, whether correlation-based or derived from Fisher information, affects the resulting curvature values, though the qualitative behavior should remain consistent. The correlation-based metric offers computational simplicity and immediate applicability to standard financial data, while the Fisher information metric provides stronger theoretical grounding at the cost of additional complexity in parameter estimation. For initial implementation, the correlation-based approach is recommended, with Fisher information serving as a refinement for subsequent iterations.

The framework's handling of dimensional consistency through non-dimensionalization ensures mathematical well-definedness, but practitioners should remain attentive to the choice of reference values when constructing ratios and the sensitivity of results to windowing parameters. The rolling window approach described in the implementation architecture introduces a trade-off between responsiveness and stability: shorter windows capture rapid changes in network structure but introduce noise, while longer windows provide smoother signals at the cost of delayed detection.

It is important to maintain appropriate epistemic humility regarding the framework's predictive capabilities. The curvature measure captures structural fragility, the degree to which perturbations are likely to amplify rather than dissipate, but does not predict the timing or direction of market moves. Empirical evidence indicates that curvature functions as a contemporaneous indicator of stress rather than a leading indicator with positive lead time (Samal et al., 2021). This limitation does not diminish the framework's value; understanding the geometric structure of financial interdependence remains valuable for risk assessment and portfolio construction even without precise predictive power.

The implementation builds on mature, well-tested libraries including GraphRicciCurvature for discrete curvature computation and POT for optimal transport calculations. This reliance on existing tools reduces implementation risk and allows practitioners to focus on the application-specific aspects of data ingestion, network construction, and result interpretation. The modular architecture separates concerns cleanly, enabling independent testing and validation of each component.

From a theoretical standpoint, the framework inherits certain limitations from its foundations. Economic processes are typically non-ergodic, as Peters (2019) has emphasized, and exhibit fundamental asymmetries between gains and losses that the geodesic framework does not fully capture. The transition from micro-level agent decisions to macro-level manifold structure involves aggregation assumptions that remain implicit rather than derived. These theoretical gaps do not preclude practical utility but should inform interpretation of results.

The conservation principles proposed (accounting identities, no-arbitrage conditions, and budget constraints) provide partial grounding for the geometric approach but fall short of the complete field equations that would fully specify dynamics. This incompleteness is acknowledged rather than concealed; the framework is offered as a practical tool for measuring network fragility rather than a complete theory of economic dynamics.

In conclusion, this whitepaper has presented a complete specification for implementing geometric fragility measurement in financial networks. The theoretical foundations connect the approach to established traditions in both physics and economics, while the implementation architecture provides a concrete path to deployment. The core hypothesis, namely that discrete

Ricci curvature is associated with financial stress and captures structural information beyond simple statistics, is empirically testable with the tools and methods described. The framework represents a practical application of differential geometric concepts to financial risk assessment, grounded in rigorous mathematics while remaining computationally tractable for real-world use.

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A Mathematical Preliminaries

A.1 Riemannian Geometry Essentials

A *Riemannian manifold* is a pair $(\mathscr{M}, g)$ where $\mathscr{M}$ is a smooth manifold and g is a smoothly varying inner product on each tangent space $T_p\mathscr{M}$.

The *Christoffel symbols* encode how vectors change under parallel transport:

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right) \tag{31}$$

The *Riemann curvature tensor* measures the failure of parallel transport to commute:

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} \tag{32}$$

The *Ricci tensor* is the trace: $R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}$

The *scalar curvature* is the full trace: $R = g^{\mu\nu} R_{\mu\nu}$

A.2 Discrete Ricci Curvature

On a weighted graph $G = (V, E, w)$, the *Ollivier-Ricci curvature* (Ollivier, 2009) of edge (x, y) is:

$$\kappa(x,y) = 1 - \frac{W_1(\mu_x, \mu_y)}{d(x,y)} \tag{33}$$

where μ_x is a probability measure on neighbors of x, W_1 is the Wasserstein-1 distance, and $d(x,y)$ is the edge weight.

Positive curvature indicates clustering; negative curvature indicates a bottleneck.

B Derivation of Two-Sector Curvature

For the metric $ds^2 = (dr^1)^2 + e^{\alpha r^1}(dr^2)^2$:

Let $g_{11} = 1$ and $g_{22} = e^{\alpha r^1}$.

Christoffel symbols:

The only non-zero derivatives are $\partial g_{22}/\partial r^1 = \alpha e^{\alpha r^1}$.

$$\Gamma^1_{22} = -\frac{1}{2g_{11}}\frac{\partial g_{22}}{\partial r^1} = -\frac{\alpha}{2}e^{\alpha r^1} \tag{34}$$

$$\Gamma^2_{12} = \frac{1}{2g_{22}}\frac{\partial g_{22}}{\partial r^1} = \frac{\alpha}{2} \tag{35}$$

Gaussian curvature:

For a metric $ds^2 = (dx)^2 + h(x)(dy)^2$, the Gaussian curvature is:

$$K = -\frac{1}{\sqrt{h}}\frac{d^2\sqrt{h}}{dx^2} \tag{36}$$

With $h = e^{\alpha r^1}$:

- $\sqrt{h} = e^{\alpha r^1/2}$
- $\frac{d\sqrt{h}}{dr^1} = \frac{\alpha}{2}e^{\alpha r^1/2}$
- $\frac{d^2\sqrt{h}}{d(r^1)^2} = \frac{\alpha^2}{4}e^{\alpha r^1/2}$

Therefore:

$$K = -\frac{1}{e^{\alpha r^1/2}} \cdot \frac{\alpha^2}{4}e^{\alpha r^1/2} = -\frac{\alpha^2}{4} \tag{37}$$

Cross-check via the Riemann tensor: With the Christoffel symbols above, the only independent component is

$$R^1_{212} = \partial_1 \Gamma^1_{22} - \Gamma^1_{22}\Gamma^2_{12} = -\frac{\alpha^2}{2}e^{\alpha r^1} + \frac{\alpha^2}{4}e^{\alpha r^1} = -\frac{\alpha^2}{4}e^{\alpha r^1}, \tag{38}$$

so that $R_{1212} = g_{11}R^1_{212} = -\frac{\alpha^2}{4}e^{\alpha r^1}$ and $K = R_{1212}/\det g = -\alpha^2/4$, confirming the result.

The curvature is constant and negative, characteristic of hyperbolic geometry. The scalar curvature is $R = 2K = -\alpha^2/2$.